



## On the use of forward kinematic models in visually guided hand position control—analysis based on ISLES model

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### Abstract

The human nervous system is equipped with a forward kinematics model, which calculates the hand positions using proprioceptive information. Since significant evidence suggests that the forward dynamics model is used for general motion control, the forward kinematics model also seems to be used for the control of visually guided reaching. However, we believe that the forward kinematics model plays no role, or only a supplemental role, in the final stages of visually guided hand position control. Instead, we propose that this is mainly handled by the inverse kinematics model. To explain the relatively large errors of the internal models in the human nervous system, Maeda et al. (Proceedings of the 1993 International Joint Conference on Neural Networks (IJCNN'93 Nagoya), 1993, pp. 1317–1320) proposed a neural network architecture called independent scalar learning elements summations (ISLES) model. We will provide evidence based on experimental results and a mathematical analysis using the ISLES model to support our hypothesis. © 2002 Published by Elsevier Science B.V.

*Keywords:* Forward kinematics model; Inverse kinematics model; Visually guided reaching; ISLES model

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## 1. Introduction

An important subject matter in neuroscience is the internal model involved in human motion control [3]. Ito proposed that the cerebellum provides forward models for a variety of controlled systems [3]. Much evidence supports the assumption that the forward dynamics model is used for motion control.

A person can estimate the position of the hands with a certain precision even when they are out of sight. It is known that the human nervous system is equipped with a forward kinematics model of the arm that calculates the hand position using proprioceptive information. Without visual feedback, a monkey loses its hand position when area five of its parietal lobe is cooled [11]. The neuronal activities of the parietal cortex corresponding to the forward kinematics model were discovered [2]. It is often supposed that the forward kinematics model that calculates the hand position using proprioceptive information operates in the posterior parietal cortex. Hereafter, we call this forward kinematics model FKMP (Forward Kinematics Model which uses Proprioceptive information). It is possible that the other forward kinematics model uses motor commands to estimate the hand position. Hereafter, we call this forward kinematics model FKMM (Forward Kinematics Model which uses Motor commands).

Since there is significant evidence supporting the use of the forward dynamics model in hand position control, it seems plausible that the forward kinematic models are also used for visually guided reaching. However, we present evidence that supports precisely the opposite conclusion:

- (1) The FKMP plays no role, or only a supplemental role, in the final stages of visually guided reaching. The human nervous system cannot freely use the forward kinematics model.
- (2) The properties of the FKMP are different from those of the FKMM, if the FKMM plays a main role in the stages.
- (3) It is highly probable that the inverse kinematics model plays the main role.

We will provide experimental evidence and theoretical predictions based on our mathematical model of sensorimotor integration to support these claims.

## 2. Background

### 2.1. Human forward and inverse kinematics models

This section presents models of the human inverse kinematics solver. Let  $\theta \in \mathbf{R}^m$  be the joint angle vector and  $x \in \mathbf{R}^n$  be the hand position/orientation vector given by the visual system. The relationship between  $x$  and  $\theta$  is expressed as  $x = f(\theta)$ , where  $f$  is a  $C^1$  class function. The Jacobian of the hand position vector is expressed as  $J(\theta) = \partial f(\theta) / \partial \theta$ . Let  $x_d$  be the desired hand position/orientation vector and  $e = x_d - x = x_d - f(\theta)$  be the hand position error vector. We consider the inverse kinematics problem that involves calculating the joint angle vector  $\theta$  satisfying  $x_d = f(\theta)$  from a desired

hand position vector  $\mathbf{x}_d$ . In this paper, a function  $\mathbf{f}^{-1}(\mathbf{x})$  that satisfies  $\mathbf{x} = \mathbf{f}(\mathbf{f}^{-1}(\mathbf{x}))$  is called an inverse kinematics function of  $\mathbf{f}(\boldsymbol{\theta})$ . The acquired model of  $\mathbf{f}^{-1}(\mathbf{x})$  in the nervous system is called an inverse kinematics model. Hereafter, we call the inverse kinematics model IKM. Let  $\Phi_{\text{im}}(\mathbf{x}) \in \mathbf{R}^m$  be the output of the inverse kinematics model. Let  $\Phi_{\text{fm}}(\boldsymbol{\theta}) \in \mathbf{R}^n$  be the output of the forward kinematics model, which approximates  $\mathbf{f}(\boldsymbol{\theta})$ .

A human can conduct a visually guided reaching motion at a certain precision even without using the visual feedback of the hand position [10]. There are two ways to solve the inverse kinematics problem without hand position error feedback. The straightforward method is through the use of the IKM. If the IKM  $\Phi_{\text{im}}(\mathbf{x}_d)$  is sufficiently precise, then an inverse kinematics solution  $\boldsymbol{\theta}_{\text{im}}$  is calculated as  $\boldsymbol{\theta}_{\text{im}} = \Phi_{\text{im}}(\mathbf{x}_d)$ . The other approach is via the use of the forward kinematics model. A number of researchers have proposed and used the method that exploits the forward model of the controlled system and have solved the control problem by using the iterative improvement technique. The methods obtain an approximate solution for the inverse problems described as  $\mathbf{x}_d = \mathbf{f}(\boldsymbol{\theta})$  by solving the following nonlinear equation:

$$\mathbf{x}_d = \Phi_{\text{fm}}(\boldsymbol{\theta}). \quad (1)$$

By using the multiple starts of the iterative procedure, Eq. (1) can usually be solved. For example, Newton's method can be used for solving Eq. (1). Let  $\boldsymbol{\theta}(k)$  be the estimated joint angle vector at step  $k$  and  $\mathbf{J}^+(\boldsymbol{\theta})$  be the pseudo-inverse matrix (Moore–Penrose's generalized inverse matrix) of  $\mathbf{J}(\boldsymbol{\theta})$ , which is calculated as  $\mathbf{J}^+(\boldsymbol{\theta}) = \mathbf{J}^T(\boldsymbol{\theta})(\mathbf{J}(\boldsymbol{\theta})\mathbf{J}^T(\boldsymbol{\theta}))^{-1}$ . By using the following iterative computation, a solution that satisfies Eq. (1) can be obtained

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \mathbf{J}^+(\boldsymbol{\theta}(k))(\mathbf{x}_d - \Phi_{\text{fm}}(\boldsymbol{\theta}(k))). \quad (2)$$

## 2.2. Independent scalar learning elements summations (ISLES) model

To explain the relatively large errors of the internal models, Maeda et al. proposed a neural network architecture called the independent scalar learning elements summations (ISLES) model [7]. Let  $\mathbf{q} = (q_1, q_2, \dots, q_u)^T \in \mathbf{R}^u$  be the input vector and  $\Phi(\mathbf{q}) \in \mathbf{R}^v$  be the output of the ISLES model. The ISLES model consists of a number of scalar learning elements  $\Phi_{s\{ij\}}(q_j)$  ( $i = 1, 2, \dots, v$ ,  $j = 1, 2, \dots, u$ ). Each element receives one scalar signal and calculates one scalar output. The  $i$ th component of the output  $\Phi_i(\mathbf{q})$  is calculated as follows:

$$\Phi_i(\mathbf{q}) = \sum_{j=1}^u \Phi_{s\{ij\}}(q_j). \quad (3)$$

The element can precisely reproduce any  $C^1$  class function with scalar input. However, the ISLES model cannot precisely reproduce all  $C^1$  class functions whose input is a vector. This limitation can explain many psychophysical properties of human beings such as Helmholtz's horopter [1], the distortion of vision space [5], the haptic horopter, and auditory alleys [9].

Let  $\Phi'(\mathbf{q})$  be the desired output signal for  $\Phi_i(\mathbf{q})$  and  $p(\mathbf{q})$  be the probability density function of  $\mathbf{q}$ . Let  $\mathcal{Q}$  be all the input space  $\mathbf{q}$  and  $\mathcal{Q}_j$  be the region, where  $z = q_j$  is satisfied. The learning results of the ISLES model can be described as follows:

$$\Phi_{s\{ij\}}(z) = E[\Phi'_i(\mathbf{q})|z = q_j] = \frac{\int_{\mathcal{Q}_j} p(\mathbf{q})\Phi'_i(\mathbf{q}) d\mathbf{q}}{\int_{\mathcal{Q}_j} p(\mathbf{q}) d\mathbf{q}}. \quad (4)$$

### 3. Psychophysical evidence: hand position control without visual feedback does not rely mainly on the FKMP

A number of the human nervous system's internal models have relatively large errors [6–8]. By using these errors, we can investigate the use of the internal models and the configuration of the control system. Based on Jäcksch's classical psychophysical experiment [4], Maeda et al. conducted psychophysical experiments to clarify the properties of the forward kinematics model that handles somatosensory information [6,7]. The experimental setup involved a subject in a darkroom who could not see his or her hand and could move a spotlight by using a joystick as shown in Fig. 1. First, the subject was asked to move the right hand to a given position. Then, the subject was asked to move the spotlight to the right-hand position as estimated by using proprioceptive information. This was achieved by controlling the joystick with the left hand without the visual feedback of the right-hand position. The spotlight position finally corresponded to the output of the forward kinematics model. Subjects usually finish one task in 15 s. It was found that a human points to a position nearer his/her body than the true hand position, as in Jäcksch's experiment, as shown in Fig. 1(b). A white square box indicates the true hand position and a black square indicates the hand position estimated from somatosensory information. An arrow indicates an error vector of FKMP.

Prablanc, Pelisson and their group conducted a number of experiments to clarify the properties of visually guided reaching with and without the use of hand position error feedback [10]. These experiments showed that a human reaches toward a point

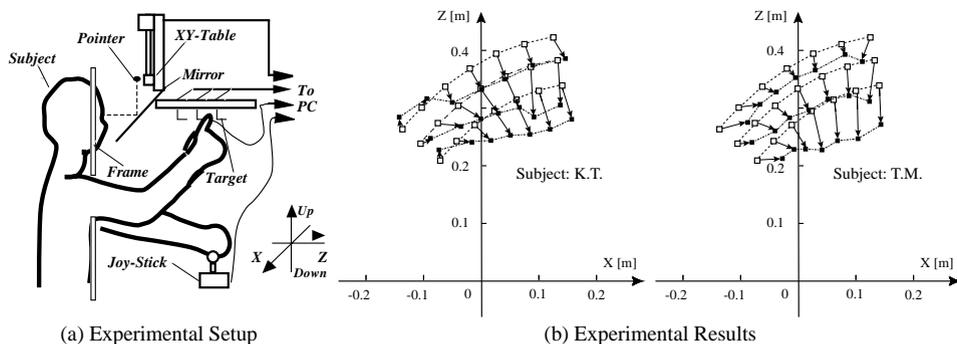


Fig. 1. Properties of FKMP: (a) experimental setup; and (b) experimental results.

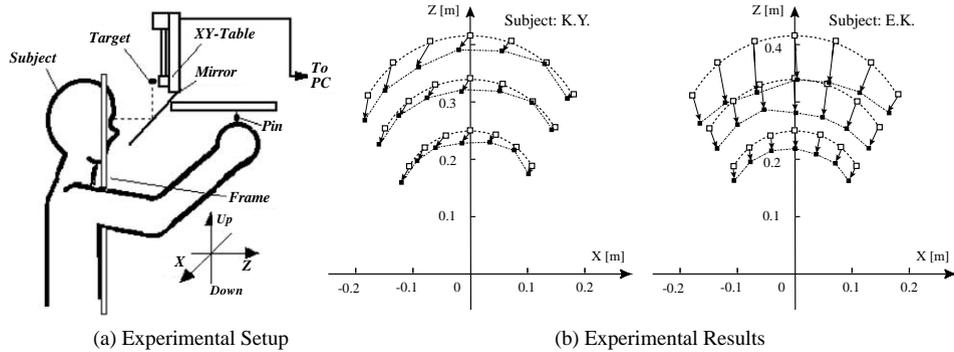


Fig. 2. Properties of reaching error without visual feedback: (a) experimental setup; and (b) experimental results.

nearer to his or her body than the target position without the hand position error feedback. Maeda improved those experiments and showed that the reaching error is expanded by the constraints imposed on the subjects as shown in Fig. 2(a) [6,7]. A subject could measure the target position only by using the rotation angles of the eyes. Fig. 2(b) shows the experimental error of visually guided reaching without hand position error feedback. A white square box indicates the target position, and a black square box indicates the real hand position as the result of reaching.

If FKMP is used for hand position control, then the real hand position should be farther from the subjects' body than the target position. However, the direction of the real error contradicts the direction of the predicted error of the inverse kinematics solution, as calculated by using the FKMP. We can conclude that the FKMP is not at all, or is only partially used in the final stage of the reaching motion without visual feedback. Furthermore, we can conclude that the properties of the FKMM are different from those of the FKMP if the FKMM plays a main role in the stages.

#### 4. Mathematical analysis based on ISLES model

There are two possible mechanisms for hand position control without using visual feedback of hand position error:

- (i) The IKM plays a main role.
- (ii) The FKMM plays a main role.

Although both these cases are possible, we suggest that the first case is more probable because the ISLES model supports the first case, as shown in this section.

##### 4.1. Forward kinematics learning by ISLES model

First, we calculated the output of FKMS, which consisted of an ISLES model. In these simulations, the visual information was described by using the bipolar latitude  $\phi$

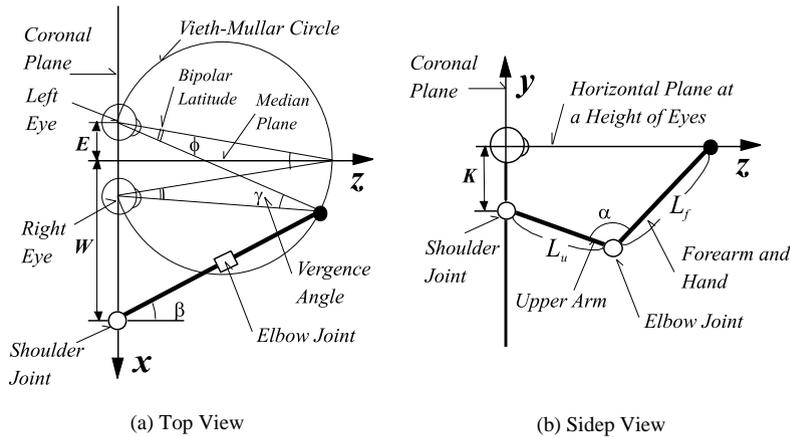


Fig. 3. Visual and motor coordinates: (a) top view; and (b) side view.

and the vergence angle  $\gamma$  of the eyes. Fig. 3 shows the visual coordinates  $\mathbf{x} = (\phi, \gamma)^T$  and the joint angle vector coordinates  $\boldsymbol{\theta} = (\alpha, \beta)^T$ . The range of  $\alpha$  was  $(-100^\circ, -95^\circ)$ . The range of  $\beta$  was  $(-50^\circ, -10^\circ)$ . The range of  $\phi$  was  $(-100^\circ, -95^\circ)$ . The range of  $\gamma$  was  $(-50^\circ, -10^\circ)$ .

We calculated the output of the FKMP  $\Phi_{\text{fm}}(\boldsymbol{\theta})$  which consists of an ISLES model using Eq. (4). The corresponding hand position on the visual coordinates  $\mathbf{x} = (\phi, \gamma)^T$  was used as the desired output signal for the FKMP as  $\Phi'_{\text{fm}}(\boldsymbol{\theta}) = \mathbf{x}$ . Let  $p_{\text{fm}}(\boldsymbol{\theta})$  be the probability density function of  $\boldsymbol{\theta}$ . We assumed that the value of  $p_{\text{fm}}(\boldsymbol{\theta})$  was constant in the region where  $\alpha$ ,  $\beta$ ,  $\phi$ , and  $\gamma$  are within the defined range and  $p_{\text{fm}}(\boldsymbol{\theta})$  was 0 in the other region. Fig. 4 shows the simulated properties of the FKMP. If we use multi-layer neural networks, then there is no significant error of the FKMP. The ISLES model can explain the properties of the FKMP.

#### 4.2. Inverse kinematics learning by ISLES model

We calculated the output of the IKM  $\Phi_{\text{im}}(\mathbf{x})$  which consists of an ISLES model using Eq. (4).  $\Phi'_{\text{im}}(\mathbf{x})$  is calculated by solving  $\mathbf{f}^{-1}(\mathbf{x})$  analytically. Let  $p_{\text{im}}(\mathbf{x})$  be the probability density function of  $\mathbf{x}$ . We assumed that the value of  $p_{\text{im}}(\mathbf{x})$  was constant in the region where  $\alpha$ ,  $\beta$ ,  $\phi$ , and  $\gamma$  are within the defined range and  $p_{\text{im}}(\mathbf{x})$  was 0 in the other region.

Fig. 5 shows the simulated reaching error by using the IKM, which consists of the ISLES model. An arrow indicates a simulated reaching error vector. The IKM, which consists of the ISLES model, can explain the properties of the errors of visually guided reaching without hand position error feedback. If the ISLES model is an appropriate model of human sensorimotor integration, then the inverse kinematics model plays the main role.

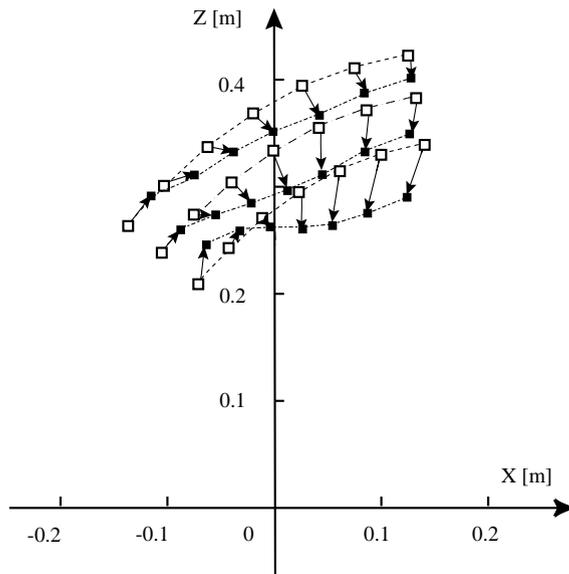


Fig. 4. Simulated error of FKMS.

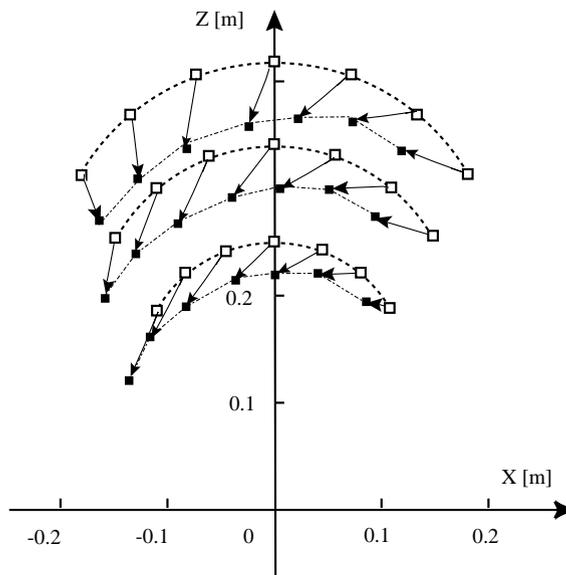


Fig. 5. Simulated reaching error.

## 5. Conclusions

In this paper, we presented experimental evidence suggesting that the forward kinematics model that calculates hand positions from proprioceptive information plays at best a supplemental role in the final stages of reaching without visual feedback. We also presented a theoretical argument to support the conclusion that the inverse kinematics model plays the main role.

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